Switching Dynamics of Neural Systems in the Presence of Multiplicative Colored Noise

Jorge F. Mejias[1], Joaquin J. Torres[1], Samuel Johnson[1],
and Hilbert J. Kappen[2]

[1] Department of Electromagnetism and Matter Physics,
University of Granada, 18071 Granada, Spain
jmejias@onsager.ugr.es
http://ergodic.ugr.es/neurofisica/neurofisica.html
[2] Department of Biophysics, Radboud University of Nijmegen
6525 EZ Nijmegen, The Netherlands

Abstract. We study the dynamics of a simple bistable system driven by multiplicative correlated noise. Such system mimics the dynamics of classical attractor neural networks with an additional source of noise associated, for instance, with the stochasticity of synaptic transmission. We found that the multiplicative noise, which performs as a fluctuating barrier separating the stable solutions, strongly influences the behaviour of the system, giving rise to complex time series and scale-free distributions for the escape times of the system. This finding may be of interest to understand nonlinear phenomena observed in real neural systems and to design bio-inspired artificial neural networks with convenient complex characteristics.

Keywords: Bistable systems, switching dynamics, multiplicative colored noise, neural up and down states.

1 Introduction

Attractor neural networks (ANN) have been a topic of high interest in the last decades. Most of the paradigmatical approaches in this field consider a number of bio-inspired elements from biological neural systems and study the computational properties of the resulting model, leading to hints and developments in neuroscience and computer algorithm design. One of the most notable models of ANN is the one proposed by Amari and Hopfield [1,2]. This model assumes a network of N binary neurons linked by connections of certain strength, also called synapses. By considering a simple learning rule based on neurophysiological data [3], this network is able to store a certain number P of patterns of activity. After this learning, the network is able to recover one of these activity patterns from an initial configuration correlated with this pattern, a property which is called *associative memory*. While the behaviour of such models is highly nonlinear, one can derive analytical solutions [4] which help to reach a better understanding for simple and hypothetical situations. For instance, if one assumes that the number of stored patterns does not scale with the size of the network, the previous model

is simplified to a bistable system. This is a common approach employed when one is interested in the dynamics of the network activity instead on its storing capacities [5]. It has been employed, for instance, to study the influence of the network topology [6], or the switching between different patterns of activity due to short-term synaptic mechanisms [7,8].

In this work, we employ a simplified bistable model, which mimics the dynamics of attractor neural networks in the limit of $P \ll N$, to study the effect of multiplicative colored noise in the dynamics of the system. Such noise resembles the stochastic nature of synaptic transmission [9] (which may be relevant to transmit information through dynamic synapses [10,11]), or other sources of colored noise which could affect the dynamics in a multiplicative way. We found that this multiplicative colored noise strongly affects the dynamics of the system, giving rise to complex time series and scale-free distributions for the escape times of the dynamics. Our results could be of interest to understand nonlinear phenomena observed in real neural systems and to design new paradigms in bio-inspired artificial neural networks.

2   Model and Results

We consider a bistable system characterized by the variable $y(t)$, which represents the level of activity of the neural network. This variable evolves according to the following discrete dynamics

$$y(t+1) = \tanh[z(t)y(t)] + \xi(t), \tag{1}$$

Here, the variable $z(t)$ is a Gaussian colored noise with mean one, standard deviation σ and correlation time τ. It represents a source of correlated noise associated with the stochasticity of the synaptic transmission, for instance. The term $\xi(t)$ is an additive Gaussian white noise of zero mean and standard deviation δ. This term takes into account other possible sources of non-multiplicative noise, and is also employed to prevent the system to be locked in the solution $y = 0$, since the fluctuations due to the multiplicative noise cannot influence the system in this point. In the following, we focus on the role of the correlation time in the dynamics, and therefore we fix the level of the fluctuations $\sigma = 50$ and $\delta = 0.1$. Similar models of bistable systems driven by multiplicative noise have been previously studied [12,13,14]. However, a detailed study of the statistics of the dynamics, which is highly relevant to understand certain collective behaviour in neural systems, is still lacking. A more complete study of the influence of these parameters will be published elsewhere [15].

From a mathematical point of view, we can see in Eq. (1) that the variable $z(t)$ represents the barrier height of our bistable system. For fixed $z < 1$, our system can be viewed as a particle in a single well potential in the presence of fluctuations (given by $\xi(t)$). Thus, $y(t)$ will be fluctuating around the only stable solution of the dynamics, $y = 0$. On the other hand, for fixed $z > 1$ we have a particle in a double well potential in the presence of fluctuations (given by $\xi(t)$). In this situation, the particle will be jumping, or switching, from one stable
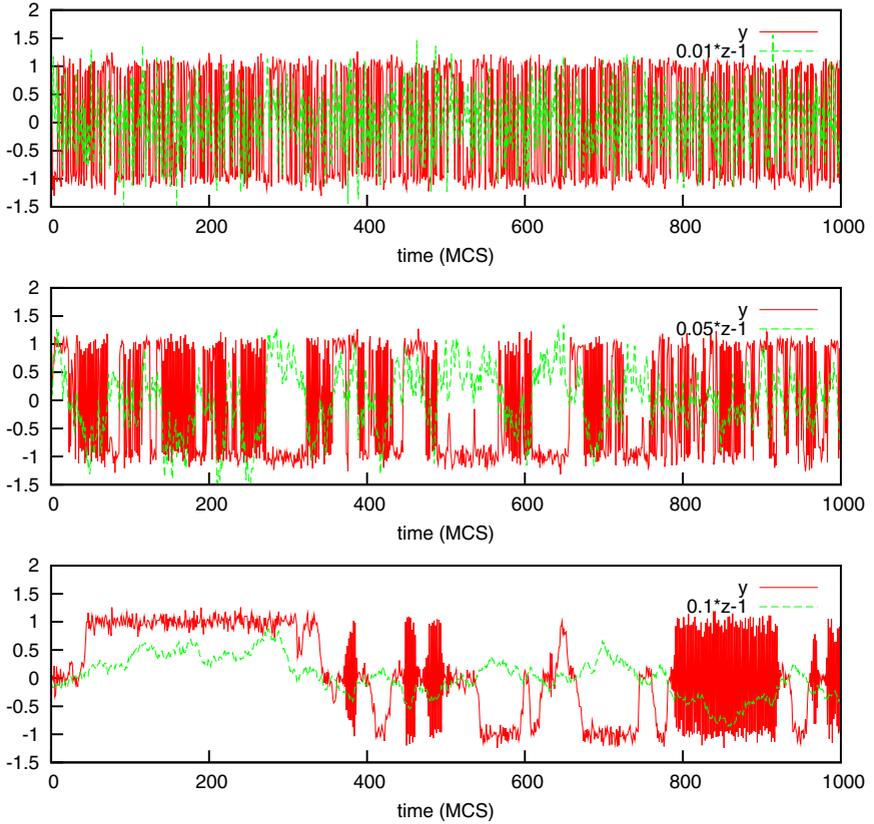
Fig. 1. (Color online) Several realizations of the time evolution of $y(t)$ and $z(t)$, for different values of the correlation time τ. The series correspond, from top to bottom, to $\tau = 0$, $\tau = 10$ and $\tau = 100$, respectively. The mean and variance of $z(t)$ have been conveniently rescaled in each case for a better visualization (see labels in each plot).

solution to the other in a stochastic manner. The fact that z has not a fixed value but it is also a fluctuating magnitude introduces a high level of complexity which is the aim of this study. For instance, we can control the characteristics of the barrier height by varying the correlation time τ. This variation has a dramatic effect in the dynamics of $y(t)$, as one can see in figure 1. The three plots in the figure shows the relation between the dynamics of $y(t)$ and $z(t)$: for $z(t) < 1$, the variable $y(t)$ rapidly fluctuates around the solution $y = 0$, and for $z(t) > 1$ we enter in the double well regime and $y(t)$ starts to swith between the symmetric stable solutions $y_+ \simeq +1$ and $y_- \simeq -1$. The switching dynamics has a strong dependence with τ, as the figure also shows. For low values of τ the switching is random and has a high frequency, but when τ is increased, some intervals of prolonged permanence in a particular solution appear. Concretely, high values of the correlation time induces drastic modifications in the probability of the
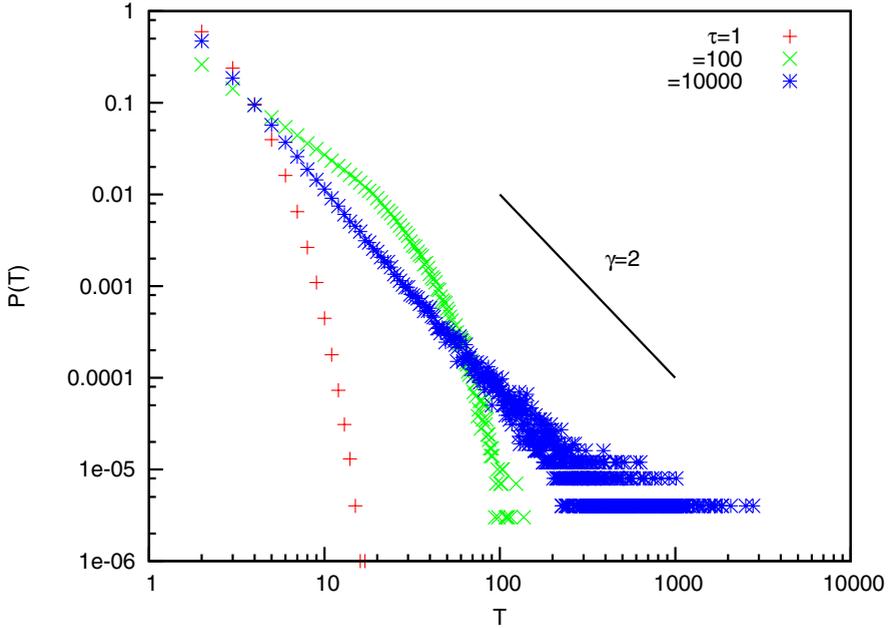
Fig. 2. Probability distribution of escape times for different values of τ. We can see that increasing of the correlation time yields the appearance of power-law distributions in escape times, which reflects the change in the statistics of the dynamics of $y(t)$. The obtained power-law distribution is defined as $P(T) \sim T^{-\gamma}$, with the exponent taking a value of $\gamma \simeq 2$.

system to jump between stable solutions after a given interval T, that is, in the probability distribution of escape times.

A more detailed study of the influence of τ in the escape time probability is shown in figure 2. One can observe that the probability distribution of the escape times is an exponential distribution for low values of τ, as it is well known. In this situation, the variable $z(t)$ behaves approximately as a white noise, and therefore $z(t)$ is continuously crossing the value $z = 1$. As a consequence of that, the double well configuration is only maintained by a very short time, and long periods of permanence in the solutions y_+ or y_- are unlikely to occur. However, when the value of τ is increased, the excursions of the variable $z(t)$ in the region of $z > 1$ become longer in time and $z(t)$ can eventually take values which are far from $z = 1$. These two factors combined allow the system to eventually stay in one of the stable solutions y_+, y_- for long periods of time, as it is shown in bottom panel of figure 1. These eventually long intervals of permanence in a double well solution are reflected in the probability distribution of escape times as a scale-free relationship, which mathematically corresponds to a power-law function $P(T) \sim T^{-\gamma}$. We found an approximate value of $\gamma \simeq 2$ for the exponent.
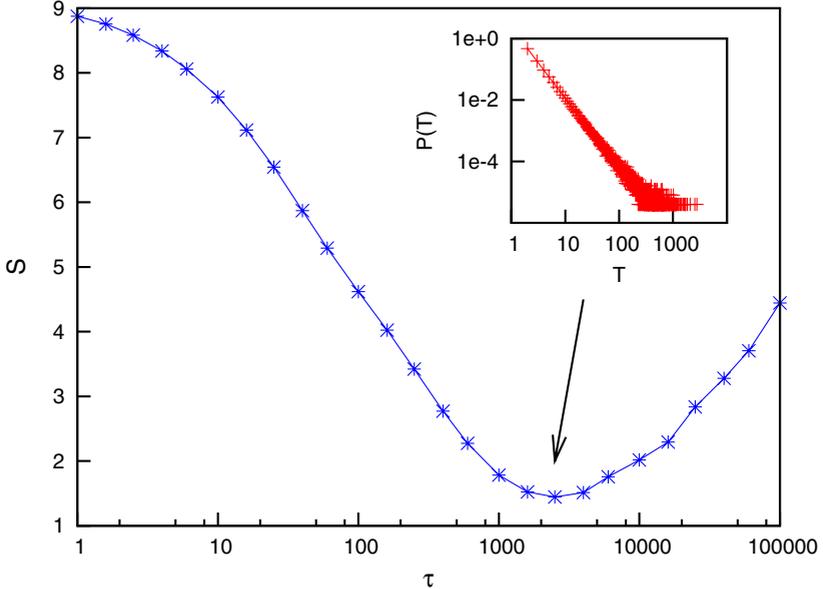
Fig. 3. The entropy function, as defined in the main text, for different values of the correlation time τ. We can observe a minimum in the entropy which approximately corresponds to the value of τ for which scale-free escape times distribution appears.

Finally, to complete the study of the dynamics of the system, we have computed a measure of the irregularity of the time series of $y(t)$, for different values of the correlation time τ. This measure is defined as an entropy of the form

$$S \equiv -\sum_x p(x) \ \log_2 \ p(x) \tag{2}$$

Here, $p(x)$ is the normalized power spectrum of the time series of $y(t)$ for a given value of τ. It is worthy to note that this entropy depends strongly on the dynamics of the system (via the power spectrum), and therefore it is influenced by the value of τ. In figure 3, we can see that the entropy reaches a minimum around the value of τ for which the scale-free distributions for the escape times appear. This is due to the fact that, in this situation, long permanence times in stable double well solutions are allowed eventually, and such permanence decreases significatively the complexity of the time series of the system. Thus, the influence of τ in the dynamics of $y(t)$ is also notorious when studying the complexity of the dynamics with multiple methods.

3   Conclusions

We have studied the dynamics of a simple bistable system under the influence of multiplicative colored noise. The results show a high impact of considering

high values of the correlation time in the dynamics of the system, and some nonlinear characteristics such as scale-free distributions and minima of entropy are found. One can think that the consequences of such complex behaviour in real neural systems, such as populations of cortical neurons connected by highly fluctuating synapses, could be relevant for different neural protocols of processing and coding of information. It has been experimentally found, for instance, that a multiplicative modulation of the noise intensity (that is, modulations in the variance of the noisy input) can be tracked much faster than additive modulations of noise (that is, modulations in the mean input) by neural populations [16]. Such fast tracking properties associated with multiplicative noises could be related with the high stimulus sensitivity present in critical systems as the one we have analyzed. On the other hand, several studies indicate the possible existence of criticality in other situations in the brain (see, for instance, [17]). Further study of these characteristics could also reveal strategies to implement the benefits obtained with this type of dynamics in bio-inspired computer algorithms.

References

1. Amari, S.: Characteristics of random nets of analog neuron-like elements. IEEE Trans. Syst. Man. Cybern. 2, 643–657 (1972)
2. Hopfield, J.J.: Neural networks and physical systems with emergent collective computational abilities. Proc. Natl. Acad. Sci. USA 79, 2554–2558 (1982)
3. Hebb, D.O.: The Organization of Behavior: A Neuropsychological Theory. Wiley, Chichester (1949)
4. Amit, D.J., Gutfreund, H., Sompolinsky, H.: Statistical mechanics of neural networks near saturation. Ann. Phys. 173, 30–67 (1987)
5. Mejias, J.F., Torres, J.J.: Maximum memory capacity on neural networks with short-term depression and facilitation. Neural Comp. 21(3), 851–871 (2009)
6. Johnson, S., Marro, J., Torres, J.J.: Functional optimization in complex excitable networks. Europhys. Lett. 83, 46006 (1–6) (2008)
7. Pantic, L., Torres, J.J., Kappen, H.J., Gielen, S.C.A.M.: Associative memmory with dynamic synapses. Neural Comput. 14, 2903–2923 (2002)
8. Torres, J.J., Cortes, J.M., Marro, J., Kappen, H.J.: Competition between synaptic depression and facilitation in attractor neural networks. Neural Comput. 19, 2739–2755 (2007)
9. Allen, C., Stevens, C.F.: An evaluation of causes for unreliability of synaptic transmission. Proc. Natl. Acad. Sci. USA 91, 10380–10383 (1994)
10. de la Rocha, J., Parga, N.: Short-term synaptic depression causes a non-monotonic response to correlated stimuli. J. Neurosci. 25(37), 8416–8431 (2005)
11. Mejias, J.F., Torres, J.J.: The role of synaptic facilitation in spike coincidence detection. J. Comp. Neurosci. 24(2), 222–234 (2008)
12. Madureira, A.J.R., Hanggi, P., Buonomano, V., Rodrigues Jr., W.A.: Escape from a fluctuating double well. Phys. Rev. E 51, 3849–3861 (1995)

13. Ya, J., Jia-rong, L., Yi-cheng, C.: A novel phase transition phenomenon in bistable system. Chin. Phys. Lett. 14, 245–247 (1997)
14. Can-Jun, W., Shi-Bo, C., Dong-Cheng, M.: Steady-state analysis of a bistable system subject to a coloured multiplicative noise and a white additive noise with coloured cross-correlated noises. Chin. Phys. 15, 1435–1440 (2006)
15. Torres, J.J., Mejias, J.F., Kappen, H.J.: Bistable neural dynamics driven by multiplicative colored noise (submitted)
16. Boucsein, C., Tetzlaff, T., Meier, R., Aertsen, A., Naundorf, B.: Dynamical response properties of neocortical neuron ensembles: multiplicative versus additive noise. J. Neurosci. 29, 1006–1010 (2009)
17. Chialvo, D.R.: Psychophysics: are our senses critical? Nat. Phys. 2, 301–302 (2006)