

FIG. 1. Phase diagram of spin-glass ferromagnet.

Figure 1: Phase portrait of SK model in the RS approximation.

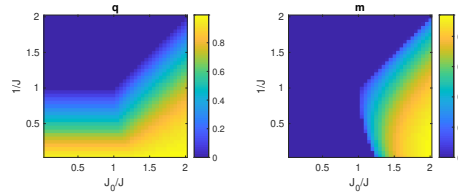


Figure 2: The solution  $q$  and  $m$  of the RS equations as a function of  $J_0/J$  and  $1/J$  ( $\beta = 1$ ).  $dx = 0.05$ .

## Phase portrait of the SK model

The phase portrait of the SK model in the replica symmetric approximation is given in fig. 1 [Sherrington and Kirkpatrick, 1975].

1. In this exercise you will reproduce this phase plot, as follows. Without loss of generality we can assume  $\beta = 1$ . Make a 2d grid equally spaced grid  $(x, y)$  with  $x = dx, 2dx, \dots, 2$  with some suitable  $dx$  and the same for  $y$ . For each  $x, y$  define  $J = 1/y, J_0 = x/y$ . Subsequently solve the RS mean field equations

$$q = 1 - \frac{1}{\sqrt{2\pi}} \int dz e^{-z^2/2} \text{sech}^2(\beta J \sqrt{q}z + \beta J_0 m)$$

$$m = \frac{1}{\sqrt{2\pi}} \int dz e^{-z^2/2} \tanh(\beta J \sqrt{q}z + \beta J_0 m)$$

using fixed point iteration. <sup>1</sup> Store the results in matrices  $q(x, y)$  and  $m(x, y)$  and display as a function of  $x, y$ . The solutions  $q(J_0/J, 1/J)$  and  $m(J_0/J, 1/J)$  should look like fig. 2, with the paramagnetic phase  $m = q = 0$ , the ferro magnetic phase  $m > 0, q > 0$  and the spin glass phase  $m = 0, q > 0$ .

2. Compute the entropy of the solution in the different phases.

<sup>1</sup>There exist standard numerical routines to compute the definite integral using Gaussian quadrature (in Matlab the routine `integral(fun, a, b)` integrates the function `fun` between `a` and `b`).

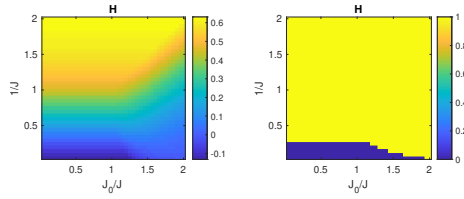


Figure 3: The entropy  $H$  of the system in the RS approximation as a function of  $J_0/J$  and  $1/J$  (left) and the thresholded value ( $H > 0$ ) (right) showing the AT line ( $\beta = 1$ ).

- (a) Use the result of MacKay exercise 31.1 to derive an expression for the entropy and the average energy from the replica symmetric free energy. The result is

$$\begin{aligned}
 H &= -\frac{\beta^2}{4} J^2 (q-1)^2 + \frac{1}{\sqrt{2\pi}} \int dz e^{-z^2/2} \log[2 \cosh(\beta J_0 m + \beta J \sqrt{q} z)] \\
 &\quad -\beta J_0 m^2 - \beta^2 J^2 q (1-q) \\
 \langle E \rangle &= -\frac{1}{2} J_0 m^2 - \frac{1}{2} \beta J^2 (1-q^2)
 \end{aligned}$$

- (b) Compute the entropy in the phase plot similar to fig. 3. Hint: I encountered problems evaluating numerically the integral  $\frac{1}{\sqrt{2\pi}} \int dz e^{-z^2/2} \log[2 \cosh(\beta J_0 m + \beta J \sqrt{q} z)]$ . I resolved it by writing  $\log 2 \cosh(f(z)) = |f(z)| + \log(1 + e^{-2|f(z)|})$  with  $f(z) = \beta J_0 m + \beta J \sqrt{q} z$ . This removes the numerical overflow of  $e^{|f(z)|}$  for large  $z$  and the underflow  $e^{-|f(z)|}$  is harmless since it evaluates to zero.

## References

- [Sherrington and Kirkpatrick, 1975] Sherrington, D. and Kirkpatrick, S. (1975). Solvable model of Spin-Glass. *Physical review letters*, 35:1792–1796.