Inleiding Machine Learning

Exercises, week 1

Exercise 1

Bishop ex. 1.1

Exercise 2

Bishop ex. 1.2

Exercise 3

(see Bishop, appendix C, eq.C.1) An $N \times M$ matrix $\mathbf{A}$ has elements A_{ij} (with i the row- and j the columnindex). The transposed matrix $\mathbf{A}^T$ has elements $(\mathbf{A}^T)_{ij} = A_{ji}$. By writing out the matrix product using index notation show that

$$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T \tag{1}$$

Hint: $\mathbf{C} = \mathbf{AB}$ corresponds to $C_{ij} = \sum_{k=1}^{M} A_{ik} B_{kj}$

Exercise 4

By repeatedly applying the product rule, show that

$$p(X, Y, Z) = p(Z|Y, X)p(Y|X)p(X) \tag{2}$$

Exercise 5

Assume $p(Y) > 0$. Two equivalent criteria for independence are:

$$p(X, Y) = p(X)p(Y) \tag{3}$$
$$p(X|Y) = p(X) \tag{4}$$

Show that (3) implies (4) and vice versa. (When does the assumption $p(Y) > 0$ come into play?)

Exercise 6

Bishop ex. 1.3

Exercise 7

Beschouw gradient descent in een kostenlandschap gegeven door $E = a_1 x^2 + a_2 y^2$ met $a_1 > a_2 > 0$.
i a) Beredeneer voor welke waarden van ϵ de gradient descent regel convergeert en divergeert.
b) Bereken de leerparameter ϵ zodanig dat de convergentie in zowel x als y richting even snel is.