Computational Neuroscience - assignments Paul 4
Kuramoto

The Kuramoto model
Under the assumption that the state of each neuron can be represented by the phase and that the effect of synaptic input can be captured in terms of sinusoidal PRC the dynamics of a network can be written as
\[ \dot{\theta}_i = \omega_i - \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_i - \theta_j), \]
here K is the coupling strength, N is the number of oscillators, \( \theta_i \) is the phase of ith oscillator and \( \omega_i \) is its frequency.

a) Derive the following mean field equation:
\[ \dot{\theta}_i = \omega_i - K r \sin(\theta_i - \Theta) \quad \text{with} \quad r \exp(i\Theta) = \frac{1}{N} \sum_{j=1}^{N} \exp(i\theta_j) \]

b) How should you interpret \( r \) and \( \Theta \)?

c) When can you find stable solutions of the mean field equation, assuming that \( r \) and \( \Theta \) are fixed?

d) Determine using simulation the solution as a function of K. Use a network of N=1000 neurons, with a oscillation frequency \( \omega \) drawn from normal distribution with zero mean and a standard deviation of 0.1.

   Step 1. Plot \( r \) and \( \Theta \) as function of time for different values of K.
   Step 2. Plot the stationary value of \( r \), using step 1 to determine how long to simulate, as a function of K.
   Step 3. There are locked oscillators whose phase is constant (note the mean frequency has been made zero without loss of generality), and those that are not locked, whose phase will vary with time (relate this to your answer to part c). Plot the fraction of locked oscillators as a function of K. Explain your findings.