Drift diffusion model

In this exercise you will generate first passage times (FPT) from the drift diffusion model. Subsequently, you will check whether the distribution of these times coincide with the FPT distribution ρ.

The continuous time diffusion process can be simulated as

$$v(t + dt) = v(t) + \mu dt + d\xi \qquad v(0) = 0$$

where μ is the mean drift and $d\xi$ is a Gaussian random variable with mean zero and standard deviation $\sigma \sqrt{dt}$. The FPT is the smallest time t that $v(t) > v_{\mathrm{th}}$.

- Generate n diffusion trajectories $v_i(t), i = 1, \ldots, n$ with $0 < t < T$ and n as large as you can. Use $dt = 0.001, \mu = 0.1, \sigma = 0.1, v_{\mathrm{th}} = 1$ and total time $T = 20$. Hint: it is very efficient in Matlab to generate all trajectories simulaneously as a vector v of length the number of trials.

- Estimate for each trial the FPT t_i. Generate a histogram of FPTs. Hint: For a given final time T it may be that some trajectories have not reached the threshold. This can be solved by increasing T, but if they are few you can set ther FPTs as $t = T$.

- Generate a plot that compares the estimated distribution of first passage times with the theoretical prediction FPT distribution ρ,

There are various contexts in which the FPT distribution can be used, such as the reaction time for decision making [Ratcliff and Mckoon, 2008] or escape times from multi-stable systems. In those situations, one wants to estimate the model parameters from observed (reaction) times. Here we will study whether we can estimate the model parameters μ, σ from our self generated data. For this we use the maximum likelihood framework. Given observed FPTs $t_i, i = 1, \ldots, n$, the log likelihood is

$$L = \sum_{i=1}^{n} \log \rho(t_i|\mu, \sigma) = n \log \frac{1}{\sigma} - \frac{3}{2} \sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} \frac{(1 - \mu t_i)^2}{2\sigma^2 t_i}$$

- Show that the maximum likelihood estimates are given by

  $$\mu = \frac{1}{\bar{t}} \qquad \sigma^2 = f - \frac{1}{\bar{t}}$$

  with $\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i$ and $f = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{t_i}$.

  Answer:

- Generate different data sets and compare the empirical estimates of μ, σ with the model values for different values of μ, σ.

Code:

References

[Ratcliff and Mckoon, 2008] Ratcliff, R. and Mckoon, G. (2008). The diffusion decision model: theory and data for two-choice decision tasks. *Neural Computation*, 20.